

Specialist Methods

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# The ACT Senior Secondary System

The ACT senior secondary system recognises a range of university, vocational or life skills pathways.

The system is based on the premise that teachers are experts in their area: they know their students and community and are thus best placed to develop curriculum and assess students according to their needs and interests. Students have ownership of their learning and are respected as young adults who have a voice.

A defining feature of the system is school-based curriculum and continuous assessment. School-based curriculum provides flexibility for teachers to address students’ needs and interests. College teachers have an opportunity to develop courses for implementation across ACT schools. Based on the courses that have been accredited by the BSSS, college teachers are responsible for developing programs of learning. A program of learning is developed by individual colleges to implement the courses and units they are delivering.

Teachers must deliver all content descriptions; however, they do have flexibility to emphasise some content descriptions over others. It is at the discretion of the teacher to select the texts or materials to demonstrate the content descriptions. Teachers can choose to deliver course units in any order and teach additional (not listed) content provided it meets the specific unit goals.

School-based continuous assessment means that students are continually assessed throughout years 11 and 12, with both years contributing equally to senior secondary certification. Teachers and students are positioned to have ownership of senior secondary assessment. The system allows teachers to learn from each other and to refine their judgement and develop expertise.

Senior secondary teachers have the flexibility to assess students in a variety of ways. For example: multimedia presentation, inquiry-based project, test, essay, performance and/or practical demonstration may all have their place. College teachers are responsible for developing assessment instruments with task specific rubrics and providing feedback to students.

The integrity of the ACT Senior Secondary Certificate is upheld by a robust, collaborative and rigorous structured consensus-based peer reviewed moderation process. System moderation involves all Year 11 and 12 teachers from public, non-government and international colleges delivering the ACT Senior Secondary Certificate.

Only students who desire a pathway to university are required to sit a general aptitude test, referred to as the ACT Scaling Test (AST), which moderates student course scores across subjects and colleges. Students are required to use critical and creative thinking skills across a range of disciplines to solve problems. They are also required to interpret a stimulus and write an extended response.

Senior secondary curriculum makes provision for student-centred teaching approaches, integrated and project-based learning inquiry, formative assessment and teacher autonomy. ACT Senior Secondary Curriculum makes provision for diverse learners and students with mild to moderate intellectual disabilities, so that all students can achieve an ACT Senior Secondary Certificate.

The ACT Board of Senior Secondary Studies (BSSS) leads senior secondary education. It is responsible for quality assurance in senior secondary curriculum, assessment and certification. The Board consists of representatives from colleges, universities, industry, parent organisations and unions. The Office of the Board of Senior Secondary Studies (OBSSS) consists of professional and administrative staff who support the Board in achieving its objectives and functions.

# ACT Senior Secondary Certificate

Courses of study for the ACT Senior Secondary Certificate:

* provide a variety of pathways, to meet different learning needs and encourage students to complete their secondary education
* enable students to develop the essential capabilities for twenty-first century learners
* empower students as active participants in their own learning
* engage students in contemporary issues relevant to their lives
* foster students’ intellectual, social and ethical development
* nurture students’ wellbeing, and physical and spiritual development
* enable effective and respectful participation in a diverse society.

Each course of study:

* comprises an integrated and interconnected set of knowledge, skills, behaviours and dispositions that students develop and use in their learning across the curriculum
* is based on a model of learning that integrates intended student outcomes, pedagogy and assessment
* outlines teaching strategies which are grounded in learning principles and encompass quality teaching
* promotes intellectual quality, establish a rich learning environment and generate relevant connections between learning and life experiences
* provides formal assessment and certification of students’ achievements.

## Underpinning beliefs

* All students are able to learn.
* Learning is a partnership between students and teachers.
* Teachers are responsible for advancing student learning.



# Learning Principles

* 1. Learning builds on existing knowledge, understandings and skills.

(Prior knowledge)

* 1. When learning is organised around major concepts, principles and significant real world issues, within and across disciplines, it helps students make connections and build knowledge structures.

(Deep knowledge and connectedness)

* 1. Learning is facilitated when students actively monitor their own learning and consciously develop ways of organising and applying knowledge within and across contexts.

(Metacognition)

* 1. Learners’ sense of self and motivation to learn affects learning.

(Self-concept)

* 1. Learning needs to take place in a context of high expectations.

(High expectations)

* 1. Learners learn in different ways and at different rates.

(Individual differences)

* 1. Different cultural environments, including the use of language, shape learners’ understandings and the way they learn.

(Socio-cultural effects)

* 1. Learning is a social and collaborative function as well as an individual one.

(Collaborative learning)

* 1. Learning is strengthened when learning outcomes and criteria for judging learning are made explicit and when students receive frequent feedback on their progress.

(Explicit expectations and feedback)

# General Capabilities

All courses of study for the ACT Senior Secondary Certificate should enable students to develop essential capabilities for twenty-first century learners. These ‘capabilities’ comprise an integrated and interconnected set of knowledge, skills, behaviours and dispositions that students develop and use in their learning across the curriculum.

The capabilities include:

* literacy
* numeracy
* information and communication technology (ICT)
* critical and creative thinking
* personal and social
* ethical behaviour
* intercultural understanding.

Courses of study for the ACT Senior Secondary Certificate should be both relevant to the lives of students and incorporate the contemporary issues they face. Hence, courses address the following three priorities. These priorities are:

* Aboriginal and Torres Strait Islander histories and cultures
* Asia and Australia’s engagement with Asia
* Sustainability.

Elaboration of these General Capabilities and priorities is available on the ACARA website at [www.australiancurriculum.edu.au](http://www.australiancurriculum.edu.au).

### Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

### Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains financial applications of Mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age. Students will also learn about the probability of certain events occurring and will therefore be well equipped to make informed decisions.

### Information and Communication Technology (ICT) Capability in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

### Critical and Creative Thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

### Personal and Social Capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

### Ethical Understanding in Mathematics

In the senior years students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical behaviour. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

### Intercultural Understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person’s environment and language.

# Cross-Curriculum Priorities

### Aboriginal and Torres Strait Islander Histories and Cultures

The senior secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students’ understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples histories and cultures.

### Asia and Australia’s Engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia’s place in the region. Through analysis of relevant data, students are provided with opportunities to further develop an understanding of the diverse nature of Asia’s environments and traditional and contemporary cultures.

### Sustainability

Each of the senior Mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Therefore, teachers are encouraged to select contexts for discussion connected with sustainability. Through analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

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# Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real-world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The major themes of Specialist Methods are calculus and statistics. They include as necessary prerequisites studies of algebra, functions and their graphs, and probability. They are developed systematically, with increasing levels of sophistication and complexity. Calculus is essential for developing an understanding of the physical world because many of the laws of science are relationships involving rates of change. Statistics is used to describe and analyse phenomena involving uncertainty and variation. The Specialist Methods course extends and develops the Mathematical Methods course with both additional content and greater depth and breadth of treatment. This is provided by more emphasis on structure and proof, by incorporating more challenging and abstract problems and the inclusion of more opportunities to develop their mathematical insight through research and exploration.

For these reasons this subject provides in-depth preparation for further studies in disciplines in which mathematics and statistics have major roles. In summary, the subject Specialist Methods is designed for students whose future pathways involve mathematical and statistical applications in a range of disciplines at the tertiary level. In addition, this course is designed for students who wish to pursue the study of mathematics itself.

For all content areas of Specialist Methods, the proficiency strands of the F-10 curriculum are still applicable and should be inherent in students’ learning of this subject. These strands are Understanding, Fluency, Problem solving and Reasoning, and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as calculating derivatives and integrals, or solving quadratic equations, and frees up working memory for more complex aspects of problem solving. The ability to transfer skills to solve problems based on a wide range of applications is a vital part of mathematics in this subject. Because both calculus and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout this subject.

Specialist Methods is structured over four units. The topics in Unit 1 build on students’ mathematical experience. The topics ‘Functions and graphs’, ‘Trigonometric functions’ and ‘Counting and probability’ all follow on from topics in the F-10 curriculum from the strands, Number and Algebra, Measurement and Geometry and Statistics and Probability. In Specialist Methods there is a progression of content and applications in all areas. For example, in Unit 2 differential calculus is introduced, and then further developed in Unit 3 where integral calculus is introduced. Unit 4 looks at topics in probability and statistics, introduces linear regression, and progresses to probability distributions and inferential statistics.

# Goals

Specialist Methods aims to develop students’:

* understanding of concepts and techniques drawn from algebra, the study of functions, calculus, probability and statistics
* ability to solve applied problems using concepts and techniques drawn from algebra, functions, calculus, probability and statistics
* reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information including ascertaining the reasonableness of solutions to problems
* capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
* capacity to choose and use technology appropriately and efficiently.

## Student Group

In Specialist Methods, there is a strong emphasis on mutually reinforcing proficiencies in Understanding, Fluency, Problem solving and Reasoning. Students gain fluency in a variety of mathematical and statistical skills, including algebraic manipulations, constructing and interpreting graphs, calculating derivatives and integrals, applying probabilistic models, estimating probabilities and parameters from data, and using appropriate technologies. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving. In order to study Specialist Methods, it is highly recommended that students complete the following topics from 10A:

* Define rational and irrational numbers, and perform operations with surds and fractional indices.
* Factorise monic and non-monic quadratic expressions, and solve a wide range of quadratic equations derived from a variety of contexts.
* Calculate and interpret the mean and standard deviation of data, and use these to compare data sets.

# Organisation of Content

Specialist Methods focuses on the development of the use of calculus and statistical analysis. The study of calculus in Specialist Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Specialist Methods develops the ability to describe and analyse phenomena involving uncertainty and variation.

Specialist Methods is organised into four units. The topics broaden students’ mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. The probability and statistics topics lead to an introduction to statistical inference.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Unit 1 | Unit 2 | Unit 3 | Unit 4 |
| Specialist  Methods | * Functions and graphs * Trigonometric functions * Counting and Probability | * Exponential functions * Arithmetic and geometric sequences and series * Introduction to differential calculus | * The logarithmic function * Further differentiation and applications * Integrals | * Simple linear regression * Discrete random variables * Continuous random variables and the normal distribution * Interval estimates for proportions |

# Unit Titles

### Unit 1

Unit 1 begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of functions and calculus. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph. The study of the trigonometric functions begins with a consideration of the unit circle using degrees and the trigonometry of triangles and its application. Radian measure is introduced, and the graphs of the trigonometric functions are examined and their applications in a wide range of settings are explored. The study of probability begins with a review of the fundamentals and the introduction to the concepts of conditional probability and independence. The study of probability and statistics allows students to further develop their counting techniques in combinatorics in Specialist Mathematics.

### Unit 2

In Unit 2 exponential functions and logarithms as their inverses are introduced and their properties and graphs examined. Arithmetic and geometric sequences and their applications are introduced and their recursive definitions applied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an ‘instantaneous rate of change’. These concepts are reinforced numerically (by calculating difference quotients), geometrically (as slopes of chords and tangents), and algebraically. This first calculus topic concludes with derivatives of polynomial functions, using simple applications of the derivative to sketch curves, calculate slopes and equations of tangents, determine instantaneous velocities, and solve optimisation problems.

### Unit 3

In Unit 3 the logarithmic function is studied in more detail. The study of calculus continues by introducing the derivatives of exponential and trigonometric functions and their applications, as well as some basic differentiation techniques and the concept of a second derivative, its meaning and applications. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The unit includes integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. Derivatives of logarithmic and exponential functions are explored.

### Unit 4

In Unit 4 simple linear regression is considered for bivariate data. Discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference. Continuous random variables are introduced and their applications examined. Probabilities associated with continuous distributions are calculated using definite integrals. In this unit students are introduced to one of the most important parts of statistics, namely statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit, inference is restricted to estimating proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations.

# Assessment

The identification of criteria within the achievement standards and assessment task types and weightings provides a common and agreed basis for the collection of evidence of student achievement.

**Assessment Criteria** (the dimensions of quality that teachers look for in evaluating student work) provide a common and agreed basis for judgement of performance against unit and course goals, within and across colleges. Over a course, teachers must use all these criteria to assess students’ performance but are not required to use all criteria on each task. Assessment criteria are to be used holistically on a given task and in determining the unit grade.

**Assessment Tasks** elicit responses that demonstrate the degree to which students have achieved the goals of a unit based on the assessment criteria. The Common Curriculum Elements (CCE) is a guide to developing assessment tasks that promote a range of thinking skills (see Appendix C). It is highly desirable that assessment tasks engage students in demonstrating higher order thinking.

**Rubrics** are constructed for individual tasks, informing the assessment criteria relevant for a particular task and can be used to assess a continuum that indicates levels of student performance against each criterion.

## Assessment Criteria

Students will be assessed on the degree to which they demonstrate an understanding of:

* concepts and techniques
* reasoning and communications.

## Assessment Task Types

|  |  |
| --- | --- |
| Suggested tasks:   * project/assignment * modelling projects * portfolio * journal * validation activity | * presentation such as a pitch, poster, vodcast, interview * practical activity such as a demonstration * test/examination * online adaptive tasks/quiz |
| Weightings in T 1.0 Units:  No task to be weighted more than 50% for a standard 1.0 unit. | |

### Additional Assessment Information

#### Requirements

* For a standard unit (1.0), students must complete a minimum of three assessment tasks and a maximum of five.
* For a half standard unit (0.5), students must complete a minimum of two and a maximum of three assessment tasks.
* Students should experience a variety of task types (test and non-test) and different modes of communication to demonstrate the Achievement Standards.
* Students are required to undertake at least one problem solving investigation task each semester. This task may be completed individually or collaboratively. They are required to plan, enquire into and draw conclusions about key unit concepts. Students may respond in forms such as modelling projects, problem solving and practical activities.
* Assessment tasks for a standard (1.0) or half-standard (0.5) unit must be informed by the Achievement Standards.

#### Advice

* It is recommended that the total component of unsupervised tasks be no greater than 30%.
* For tasks completed in unsupervised conditions, schools need to have mechanisms to uphold academic integrity, for example, student declaration, plagiarism software, oral defence, interview, other validation tasks

# Achievement Standards

Years 11 and 12 Achievement Standards are written for A/T courses. A single Achievement Standard is written for M courses.

A Year 12 student in any unit is assessed using the Year 12 Achievement Standards. A Year 11 student in any unit is assessed using the Year 11 Achievement Standards. Year 12 Achievement Standards reflect higher expectations of student achievement compared to the Year 11 Achievement Standards. Years 11 and 12 Achievement Standards are differentiated by cognitive demand, the number of dimensions and the depth of inquiry.

An Achievement Standard cannot be used as a rubric for an individual assessment task. Assessment is the responsibility of the college. Student tasks may be assessed using rubrics or marking schemes devised by the college. A teacher may use the Achievement Standards to inform development of rubrics. The verbs used in Achievement Standards may be reflected in the rubric. In the context of combined Years 11 and 12 classes, it is best practice to have a distinct rubric for Years 11 and 12. These rubrics should be available for students prior to completion of an assessment task so that success criteria are clear.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Achievement Standards for Mathematics T Course – Year 11 | | | | | |
|  | A student who achieves an **A** grade typically | A student who achieves a **B** grade typically | A student who achieves a **C** grade typically | A student who achieves a **D** grade typically | A student who achieves an **E** grade typically |
| Concepts and Techniques | * critically applies mathematical concepts in a variety of complex contexts to routine and non-routine problems | * applies mathematical concepts in a variety of contexts to routine and non-routine problems | * applies mathematical concepts in some contexts to routine and non-routine problems | * applies simple mathematical concepts in limited contexts to routine problems | * applies simple mathematical concepts in structured contexts |
| * selects and applies advanced mathematical techniques to solve complex problems in a variety of contexts | * selects and applies mathematical techniques to [solve](http://www.australiancurriculum.edu.au/Glossary?a=&t=Solve) routine and [non-routine](http://www.australiancurriculum.edu.au/Glossary?a=&t=Non-routine) problems in a variety of contexts | * applies mathematical techniques to [solve](http://www.australiancurriculum.edu.au/Glossary?a=&t=Solve) [routine and non-routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in some contexts | * applies simple mathematical techniques to solve [routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in limited contexts | * uses simple mathematical techniques to solve routine problems in [structured](http://www.australiancurriculum.edu.au/Glossary?a=&t=Structured) contexts |
| * constructs, selects and applies complex mathematical models to routine and non-routine problems in a variety of contexts | * selects and applies mathematical models to routine and [non-routine](http://www.australiancurriculum.edu.au/Glossary?a=&t=Non-routine) problems to a variety of contexts | * applies mathematical models to [routine and non-routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in some contexts | * applies simple mathematical models to routine problems in limited contexts | * demonstrates limited familiarity with mathematical models in [structured](http://www.australiancurriculum.edu.au/Glossary?a=&t=Structured) contexts |
| * uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts | * uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts | * uses digital technologies appropriately to solve routine and non-routine problems in some contexts | * uses digital technologies appropriately to solve routine problems in limited contexts | * uses digital technologies to solve routine problems in structured contexts |
| Reasoning and Communications | * represents complex mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts | * represents mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems a variety of contexts | * represents mathematical concepts in numerical, graphical and symbolic form to some [routine and some non-routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in some contexts | * represents simple mathematical concepts in numerical, graphical or symbolic form in [routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in limited contexts | * represents simple mathematical concepts in numerical, graphical or symbolic form in [structured](http://www.australiancurriculum.edu.au/Glossary?a=&t=Structured) contexts |
| * communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are succinct and well-reasoned, using appropriate and accurate language | * communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are clear and reasoned, using appropriate and accurate language | * [communicates](http://www.australiancurriculum.edu.au/Glossary?a=&t=Communicates) mathematical judgements and arguments in oral, written and/or multimodal forms, using appropriate and accurate language | * [communicates](http://www.australiancurriculum.edu.au/Glossary?a=&t=Communicates) simple mathematical judgements or arguments in oral, written and/or multimodal forms, with some use of appropriate language | * [communicates](http://www.australiancurriculum.edu.au/Glossary?a=&t=Communicates) simple mathematical information in oral, written and/or multimodal forms, with limited use of appropriate language |
| * evaluates the reasonableness of solutions to routine and non-routine problems in a variety of contexts | * analyses the reasonableness of solutions to routine and [non-routine](http://www.australiancurriculum.edu.au/Glossary?a=&t=Non-routine) problems | * explains the reasonableness of solutions to some [routine and non-routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) | * describes the appropriateness of solutions to [routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) | * identifies solutions to [routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in structured contexts |
| * reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively | * reflects on their own thinking and analyses planning, time management, use of appropriate strategies to work independently and collaboratively | * reflects on their own thinking and explains planning, time management, use of appropriate strategies to work independently and collaboratively | * reflects on their own thinking with some reference to planning, time management, use of appropriate strategies to work independently and collaboratively | * reflects on their own thinking with little or no reference to planning, time management, use of appropriate strategies to work independently and collaboratively |
| * evaluates the potential of Mathematics to generate knowledge in the public good | * analyses the potential of Mathematics to generate knowledge in the public good | * explains the potential of Mathematics to generate knowledge in the public good | * describes the potential of Mathematics to generate knowledge in the public good | * identifies some ways in which Mathematics is used to generate knowledge in the public good |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Achievement Standards for Mathematics T Course – Year 12 | | | | | |
|  | A student who achieves an **A** grade typically | A student who achieves a **B** grade typically | A student who achieves a **C** grade typically | A student who achieves a **D** grade typically | A student who achieves an **E** grade typically |
| Concepts and Techniques | * critically and creatively applies mathematical concepts in a variety of complex contexts to routine and non-routine problems | * critically applies mathematical concepts in a variety of contexts to routine and non-routine problems | * applies mathematical concepts in some contexts to routine and non-routine problems | * applies simple mathematical concepts in limited contexts to routine problems | * applies simple mathematical concepts in structured contexts |
| * synthesises information to select and apply mathematical techniques to solve complex problems in a variety of contexts | * analyses information to [select](http://www.australiancurriculum.edu.au/Glossary?a=&t=Select) and [apply](http://www.australiancurriculum.edu.au/Glossary?a=&t=Apply) mathematical techniques to [solve](http://www.australiancurriculum.edu.au/Glossary?a=&t=Solve) routine and [non-routine](http://www.australiancurriculum.edu.au/Glossary?a=&t=Non-routine) problems in a variety of contexts | * selects and applies mathematical techniques to [solve](http://www.australiancurriculum.edu.au/Glossary?a=&t=Solve) [routine and some non-routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in some contexts | * applies simple mathematical techniques to solve [routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in limited contexts | * uses simple mathematical techniques to solve routine problems in [structured](http://www.australiancurriculum.edu.au/Glossary?a=&t=Structured) contexts |
| * constructs, selects and applies mathematical models to a variety of contexts in routine and non-routine problems | * selects and applies mathematical models to routine and [non-routine](http://www.australiancurriculum.edu.au/Glossary?a=&t=Non-routine) problems in a variety of contexts | * applies mathematical models to [routine and non-routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in some contexts | * applies simple mathematical models to routine problems in limited contexts | * demonstrates limited familiarity with mathematical models to solve routine problems in [structured](http://www.australiancurriculum.edu.au/Glossary?a=&t=Structured) contexts |
| * uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts | * uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts | * uses digital technologies appropriately to solve routine and non-routine problems in a variety of contexts | * uses digital technologies appropriately to solve routine problems in limited contexts | * uses digital technologies to solve routine problems in structured contexts |
| Reasoning and Communications | * represents complex mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts | * represents mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts | * represents mathematical concepts in numerical, graphical and symbolic form in some [routine and non-routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in some contexts | * represents simple mathematical concepts in numerical, graphical or symbolic form in [routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in structured contexts | * represents simple mathematical concepts in numerical, graphical or symbolic form in in simple problems in [structured](http://www.australiancurriculum.edu.au/Glossary?a=&t=Structured) contexts |
| * communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are succinct and reasoned, using appropriate and accurate language | * communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are clear and reasoned, using appropriate and accurate language | * [communicates](http://www.australiancurriculum.edu.au/Glossary?a=&t=Communicates) mathematical judgements and arguments in oral, written and/or multimodal forms, using appropriate and accurate language | * [communicates](http://www.australiancurriculum.edu.au/Glossary?a=&t=Communicates) simple mathematical judgements or arguments in oral, written and/or multimodal forms, with some use of appropriate language | * [communicates](http://www.australiancurriculum.edu.au/Glossary?a=&t=Communicates) simple mathematical information in oral, written and/or multimodal forms, with limited use of appropriate language |
| * evaluates the solutions to routine and non-routine problems in a variety of contexts | * analyses the solutions to routine and [non-routine](http://www.australiancurriculum.edu.au/Glossary?a=&t=Non-routine) problems in some contexts | * explains solutions to some [routine and non-routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in some contexts | * describes solutions to [routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in limited contexts | * identifies solutions to [routine problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) in structured contexts |
| * evaluates methods and models for their strengths and limitations when developing solutions to routine and non-routine problems | * analyses strengths and limitations of models used when developing solutions to routine and non-routine problems | * explains strengths and limitations of models used when developing solutions to some routine and non-routine [problems](http://www.australiancurriculum.edu.au/Glossary?a=&t=Routine%20problems) | * describes strengths or limitations of simple models when solving routine problems | * identifies strengths or limitations of simple models in relation to routine problems |
| * reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively | * reflects on their own thinking and analyses planning, time management, use of appropriate strategies to work independently and collaboratively | * reflects on their own thinking and explains planning, time management, use of appropriate strategies to work independently and collaboratively | * reflects on their own thinking with some reference to planning, time management, use of appropriate strategies to work independently and collaboratively | * reflects on their own thinking with little or no reference to planning, time management, use of appropriate strategies to work independently and collaboratively |
| * evaluates the potential of Mathematics to generate knowledge in the public good | * analyses the potential of Mathematics to generate knowledge in the public good | * explains the potential of Mathematics to generate knowledge in the public good | * describes the potential of Mathematics to generate knowledge in the public good | * identifies some ways in which Mathematics is used to generate knowledge in the public good |

# Unit 1: Specialist Methods Value: 1.0

Unit 1a: Specialist Methods Value: 0.5

Unit 1b: Specialist Methods Value: 0.5

Students are expected to study the accredited semester 1.0 unit unless enrolled in a 0.5 unit due to late entry or early exit in a semester.

## Specific Unit Goals

By the end of this unit, students:

* understand the concepts and techniques in algebra, functions, graphs, trigonometric functions and probability
* solve problems using algebra, functions, graphs, trigonometric functions and probability
* apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions and probability
* interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
* communicate their arguments and strategies when solving problems.

## Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

## Topic 1: Functions and graphs

#### Functions:

* understand the concept of a function as a mapping between sets, and as a rule or a formula that defines one variable quantity in terms of another
* use function notation, domain and range, independent and dependent variables
* understand the concept of the graph of a function
* recognise odd and even functions
* examine translations and the graphs of and
* examine dilations and the graphs of and
* recognise the distinction between functions and relations, and the vertical line test.

*NB: The material on functions needs to be taught at the appropriate time throughout the delivery of Topic 1. It is envisaged that function ideas, definitions, and notation are taught concurrently with the concepts covered in Topic 1.*

#### Lines and linear relationships

* examine examples of direct proportion and linearly related variables
* recognise features of the graph of , including its linear nature, its intercepts and its slope or gradient
* find the equation of a straight line given sufficient information; parallel and perpendicular lines
* solve linear equations and inequations
* geometrical applications of the linear function
* determine the coordinates of the point dividing a line in any given ratio – internally and externally
* calculate distances between points, and perpendicular distances from points to lines.

#### Quadratic relationships:

* examine examples of quadratically related variables
* factorisation techniques including common factor, trinomials, difference of squares and cubes and grouping in pairs
* recognise features of the graphs of , and , including their parabolic nature, turning points, axes of symmetry and intercepts
* solve quadratic equations using the quadratic formula and by completing the square
* find the equation of a quadratic given sufficient information
* find turning points and zeros of quadratics and understand the role of the discriminant
* determine the nature of the roots of the quadratic considering the discriminant

#### Powers and polynomials:

* recognise features of the graphs of for , including shape, and behaviour as and
* identify the coefficients, leading term, constant term and the degree of a polynomial
* expand quadratic and cubic polynomials from factors
* recognise features of the graphs of cubics and higher-order polynomials, including shape, intercepts (including repeated factors) and behaviour as and
* Use division of polynomials and remainder and factor theorems to factorise higher-order polynomials
* solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained.
* model real world problems using polynomial functions

#### Inverse proportion:

* examine examples of inverse proportion
* recognise features of the graphs of and , including their hyperbolic shapes, and their asymptotes.
* Examine functions with oblique asymptotes (eg and further rational functions (eg )

#### Graphs of relations:

* recognise features of the graphs of and , including their circular shapes, their centres and their radii
* recognise features of the graph of including its parabolic shape and its axis of symmetry.

## Topic 2: Trigonometric functions

#### Cosine and sine rules:

* review sine, cosine and tangent as ratios of side lengths in right-angled triangles
* understand the unit circle definition of and and periodicity using degrees
* examine the relationship between the angle of inclination of a line and the gradient of that line
* establish and use the sine and cosine rules and the formula for the area of a triangle.
* examine the ambiguous case

#### Circular measure and radian measure:

* define and use radian measure and understand its relationship with degree measure
* calculate lengths of arcs and areas of sectors and segments in circles.

#### Trigonometric functions:

* understand the unit circle definition of and and periodicity using radians
* recognise the exact values of at integer multiples of
* recognise the graphs of on extended domains
* examine amplitude changes and the graphs of and
* examine period changes and the graphs of , and
* examine phase changes and the graphs of and as well as the relationships and
* generalise observed relationships between trigonometric functions (such as the above) as algebraic identities
* prove and apply the angle sum and difference identities
* identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems
* solve equations and inequations involving trigonometric functions using technology, and algebraically using trigonometric identities (including the Pythagorean Identity and reciprocal ratios).

## Topic 3: Counting and probability

#### Combinations:

* understand the notion of a combination as an unordered set of objects taken from a set of distinct objects
* use the notation and the formula for the number of combinations of objects taken from a set of distinct objects
* expand for small positive integers
* recognise the numbers as binomial coefficients, i.e., as coefficients in the expansion of

* use Pascal’s triangle and its properties.

#### Language of events and sets:

* review the concepts and language of outcomes, sample spaces and events as sets of outcomes
* use set language and notation for events, including (or for the complement of an event for the intersection of events and , and for the union, and recognise mutually exclusive events
* use everyday occurrences to illustrate set descriptions and representations of events, and set operations.
* solve problems using Venn Diagrams

#### Review of the fundamentals of probability:

* review probability as a measure of ‘the likelihood of occurrence’ of an event
* review the probability scale: for each event with if is an impossibility and if is a certainty
* review the rules: and
* use relative frequencies obtained from data as point estimates of probabilities.

#### Conditional probability and independence:

* understand the notion of a conditional probability and recognise and use language that indicates conditionality
* use the notation and the formula *P*(*A* ∩ *B*) = *P*(*A*|*B*)*P*(*B*)
* understand the notion of independence of an event from an event , as defined by
* use probability trees to evaluate conditional probability
* establish and use the formula for independent events and , and recognise the symmetry of independence
* use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events.

## A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learningis what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

## Assessment

Refer to pages 10-12.

# Unit 2: Specialist Methods Value: 1.0

Unit 2a: Specialist Methods Value: 0.5

Unit 2b: Specialist Methods Value: 0.5

Students are expected to study the accredited semester 1.0 unit unless enrolled in a 0.5 unit due to late entry or early exit in a semester.

## Specific Unit Goals

By the end of this unit, students:

* understand the concepts and techniques used in algebra, sequences and series, functions, graphs and calculus
* solve problems in algebra, sequences and series, functions, graphs and calculus
* apply reasoning skills in algebra, sequences and series, functions, graphs and calculus
* interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems
* communicate arguments and strategies when solving problems.

## Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

## Topic 1: Exponential functions

#### Indices and the index laws:

* review indices (including fractional and negative indices) and the index laws
* use radicals and convert to and from fractional indices
* understand and use scientific notation and significant figures
* solve problems involving very large and very small numbers
* solve simple indicial equations and inequations
* define logarithms as the inverses of exponentials: is equivalent to
* solve equations involving indices using logarithms

#### Exponential functions:

* establish and use the algebraic properties of exponential functions
* sketch, and hence recognise the qualitative features of, the graph of including asymptotes, and of its translations ( and ) and dilations ()
* identify contexts suitable for modelling by exponential functions and use them to solve practical problems
* solve equations and inequations involving exponential functions using technology, and algebraically in simple cases.

## Topic 2 Arithmetic and geometric sequences and series

#### General sequences and number patterns

* consider the role of patterns in defining mathematics generally
* consider a range of types of number patterns and ways of defining and describing them (including for example Fibonacci, triangular and perfect numbers)
* use difference equations to determine the algebraic rule for a number pattern
* Use sigma notation for series

#### Arithmetic sequences:

* recognise and use the recursive definition of an arithmetic sequence:
* prove and use the formula for the general term of an arithmetic sequence and recognise its linear nature
* use arithmetic sequences in contexts involving discrete linear growth or decay, including simple interest
* establish and use the formulae for the sum of the first terms of an arithmetic sequence.

#### Geometric sequences:

* recognise and use the recursive definition of a geometric sequence:
* use the formula for the general term of a geometric sequence and recognise its exponential nature
* understand the limiting behaviour as of the terms in a geometric sequence and its dependence on the value of the common ratio
* establish and use the limiting sum of a geometric sequence
* establish and use the formula for the sum of the first terms of a geometric sequence
* use geometric sequences in contexts involving geometric growth or decay, including compound interest and the determination of half-lives

## Topic 3: Introduction to differential calculus

#### Rates of change:

* interpret the difference quotient as the average rate of change of a function
* use the Leibniz notation and for changes or increments in the variables and
* use the notation for the difference quotient where
* interpret the ratios and as the slope or gradient of a chord or secant of the graph of .

#### The concept of the derivative:

* consider continuity of functions and types of discontinuity
* explore limits of functions from the left and from the right
* examine the behaviour of the difference quotient as as an informal introduction to the concept of a limit
* define the derivative as
* use the Leibniz notation for the derivative: and the correspondence where
* consider examples of rates of change, eg of flow from different shaped vessels, and sketch graphs describing these rates
* interpret the derivative as the instantaneous rate of change
* compare average and instantaneous rates of change
* interpret the derivative as the slope or gradient of a tangent line of the graph of , and explore the relationship between the graphs of f(x) and f’(x)

#### Computation of derivatives:

* estimate numerically the value of a derivative, for simple power functions
* examine examples of variable rates of change of non-linear functions
* establish the formula for positive integers by expanding or by factorising
* extend the formula to apply for all rational n.

#### Properties of derivatives:

* recognise whether or not a function is differentiable based on understanding the conditions for differentiability of a function
* understand the concept of the derivative as a function
* recognise and use linearity properties of the derivative
* calculate derivatives of polynomials and other linear combinations of power functions.
* understand where stationary points occur and their nature from graphs of derivative functions and vice versa

#### Applications of derivatives:

* find instantaneous rates of change
* find the slope of a tangent and normal and the equation of the tangent and normal
* construct and interpret position-time graphs, with velocity as the slope of the tangent
* sketch curves associated with polynomials; find stationary points, and local and global maxima and minima; and examine behaviour as and
* solve optimisation problems arising in a variety of contexts involving polynomials on finite interval domains.
* apply calculus techniques to straight line motion graphs including those of position time, velocity and acceleration

#### Anti-derivatives:

* calculate anti-derivatives of polynomial functions and apply to a variety of contexts including motion in a straight line.

## A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learningis what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

## Assessment

Refer to pages 10-12.

# Unit 3: Specialist Methods Value: 1.0

Unit 3a: Specialist Methods Value: 0.5

Unit 3b: Specialist Methods Value: 0.5

Students are expected to study the accredited semester 1.0 unit unless enrolled in a 0.5 unit due to late entry or early exit in a semester.

## Specific Unit Goals

By the end of this unit, students:

* understand the concepts and techniques in calculus, probability and statistics
* solve problems in calculus, probability and statistics
* apply reasoning skills in calculus, probability and statistics
* interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
* communicate their arguments and strategies when solving problems.

## Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

## Topic 1: The logarithmic function

#### Logarithmic functions:

* review the definition and algebraic properties of logarithms
* interpret and use logarithmic scales such as decibels in acoustics, the Richter Scale for earthquake magnitude, octaves in music, pH in chemistry
* recognise the qualitative features of the graph of ( including asymptotes, and of its translations and
* solve equations involving logarithmic functions algebraically and graphically
* use logarithmic functions to model and solve practical problems

## Topic 2: Further differentiation and applications

#### Differentiation rules:

* understand and use the product and quotient rules
* understand the notion of composition of functions and use the chain rule for determining the derivatives of composite functions

#### Exponential functions:

* estimate the limit of as using technology, for various values of , and thus recognise the limit as the gradient of at (0,1)
* recognise that is the unique number for which the above limit is 1
* establish and use the formulae and
* use exponential functions and their derivatives to solve practical problems.

#### Logarithmic functions:

* define the natural logarithm
* recognise and use the inverse relationship of the functions and
* establish and use the formulae and
* establish and use the formulae and
* use logarithmic functions and their derivatives and integrals to solve practical problems.

#### Trigonometric functions:

* establish the formulas by numerical estimations of the limits and informal proofs based on geometric constructions
* use trigonometric functions and their derivatives to solve practical problems.

#### Further Differentiation:

* apply the product, quotient and chain rule to differentiate functions such as , , , , , and
* determine the derivatives of the reciprocal trigonometric functions (sec, cosec, cot)

#### The second derivative and applications of differentiation:

* use the increments formula: to estimate the change in the dependent variable resulting from changes in the independent variable
* understand the concept of the second derivative as the rate of change of the first derivative function
* recognise acceleration as the second derivative of position with respect to time
* understand the concepts of concavity and points of inflection and their relationship with the second derivative
* understand and use the second derivative test for finding local maxima and minima
* sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection
* solve optimisation problems from a wide variety of fields using first and second derivatives.

## Topic 3: Integrals

#### Anti-differentiation:

* recognise anti-differentiation as the reverse of differentiation
* use the notation for anti-derivatives or indefinite integrals
* establish and use the formula for
* establish and use the formula
* establish and use the formula
* recognise the integrals of the derivatives of trigonometric functions (including reciprocal functions)
* recognise and use linearity of anti-differentiation
* determine indefinite integrals of the form
* identify families of curves with the same derivative function
* determine given nd an initial condition
* determine displacement given velocity in linear motion problems.

#### Definite integrals:

* examine the area problem, and use sums of the form to estimate the area under the curve
* interpret the definite integral as area under the curve if
* recognise the definite integral as a limit of sums of the form
* interpret as a sum of signed areas
* recognise and use the additivity and linearity of definite integrals.

#### Fundamental theorem:

* understand the concept of the signed area function
* understand and use the theorem: , and illustrate its proof geometrically
* understand the formula and use it to calculate definite integrals.

#### Applications of integration:

* calculate the area bounded by a curve and either axis
* calculate total change by integrating instantaneous or marginal rate of change
* calculate the area between curves in simple cases
* determine positions given acceleration and initial values of position and velocity
* calculate volumes of solids of revolution formed by rotating simple regions around the x axis.

## A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learningis what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

## Assessment

Refer to pages 10-12.

# Unit 4: Specialist Methods Value: 1.0

Unit 4a: Specialist Methods Value: 0.5

Unit 4b: Specialist Methods Value: 0.5

Students are expected to study the accredited semester 1.0 unit unless enrolled in a 0.5 unit due to late entry or early exit in a semester.

## Specific Unit Goals

By the end of this unit, students:

* understand the concepts and techniques in calculus, probability and statistics
* solve problems in calculus, probability and statistics
* apply reasoning skills in calculus, probability and statistics
* interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
* communicate their arguments and strategies when solving problems.

## Content Descriptions

**Note:** this Unit provides opportunities to address Aboriginal and Torres Strait Islander perspectives and AITSL standards

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

## Topic 1: Simple Linear Regression

* investigate scatter plots of two variables
* apply simple linear regression of y on x () using method of least squares, including Interpolation and Extrapolation

|  |
| --- |
|  |
| Where: and |

* compute correlation coefficient (at least Pearson’s Method) and coefficient of determination
* use technology to fit line of best fit eg Excel, Geogebra, Calculator etc
* use real-life data (possible sources are ABS or AIHW (<http://www.aihw.gov.au/data/)>)
* differentiate between correlation and causation.

## Topic 2: Discrete random variables

#### General discrete random variables:

* understand the concepts of a discrete random variable and its associated probability function, and their use in modelling data
* use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable
* recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes
* examine examples of non-uniform discrete random variables, for example Poisson and Hypergeometric distribution
* recognise the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases
* recognise the variance and standard deviation of a discrete random variable as a measures of spread, and evaluate them in simple cases
* calculate means and variances of linear combinations of random variables (e.g. (E(aX+b)=aE(X)+b, , etc)
* use discrete random variables and associated probabilities to solve practical problems

#### Bernoulli distributions:

* use a Bernoulli random variable as a model for two-outcome situations
* identify contexts suitable for modelling by Bernoulli random variables
* recognise the mean and variance of the Bernoulli distribution with parameter
* use Bernoulli random variables and associated probabilities to model data and solve practical problems.

#### Binomial distributions:

* understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of ‘successes’ in independent Bernoulli trials, with the same probability of success in each trial
* identify contexts suitable for modelling by binomial random variables
* determine and use the probabilities associated with the binomial distribution with parameters and p ; note the mean and variance of a binomial distribution
* use binomial distributions and associated probabilities to solve practical problems, such as Markov Chains
* model real-life data, drawing inferences from specific to general

## Topic 3: Continuous random variables and the normal distribution

#### General continuous random variables:

* use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable
* understand the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts
* recognise the expected value, variance and standard deviation of a continuous random variable and evaluate them in simple cases
* understand the effects of linear changes of scale and origin on the mean and the standard deviation.

#### Normal distributions:

* identify contexts such as naturally occurring variation that are suitable for modelling by normal random variables
* recognise features of the graph of the probability density function of the normal distribution with mean and standard deviation and the use of the standard normal distribution
* calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve practical problems.
* calculate interval estimate of the mean (e.g. 95% confidence limits)
* use Normal approximation to Binomial Distribution for np>5 and nq>5, taking into account correction for continuity

## Topic 4: Interval estimates for proportions

#### Random sampling:

* understand the concept of a random sample
* discuss sources of bias in samples, and procedures to ensure randomness
* use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli.

#### Sample proportions:

* understand the concept of the sample proportion as a random variable whose value varies between samples, and the formulas for the mean and standard deviation of the sample proportion
* examine the approximate normality of the distribution of for large samples
* simulate repeated random sampling, for a variety of values of *p* and a range of sample sizes, to illustrate the distribution of and the approximate standard normality of where the closeness of the approximation depends on both *n* and *p.*

#### Confidence intervals for proportions:

* the concept of an interval estimate for a parameter associated with a random variable
* use the approximate confidence interval as an interval estimate for , where is the appropriate quantile for the standard normal distribution
* define the approximate margin of error and understand the trade-off between margin of error and level of confidence
* use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain .

## A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learningis what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

## Assessment

Refer to pages 10-12.

# Appendix A – Implementation Guidelines

## Evaluation of Previous Course

This Specialist Methods course integrates the Australian Curriculum.

## Available course patterns

A standard 1.0 value unit is delivered over at least 55 hours. To be awarded a course, students must complete at least the minimum units over the whole minor, major, major/minor or double major course.

|  |  |
| --- | --- |
| Course | Number of standard units to meet course requirements |
| Minor | Minimum of 2 units |
| Major | Minimum of 3.5 units |

It is recommended that Units 1-4 are studied sequentially.

### Prerequisites for the course or units within the course

Content sequencing and delivery within and across semester units may be varied at the discretion of the Executive Teacher of Mathematics as per BSSS requirements.

Relationship to other courses

This course can be studied in conjunction with Specialist Mathematics Options (integrating the Australian Curriculum) to form a major minor or double major in Specialist Mathematics, according to the requirements in the Specialist Mathematics course document.

Students who complete a major in Specialist Methods and fewer than 2.0 units of Specialist Mathematics Options will include these units in a major in Specialist Mathematics.

Students studying Specialist Mathematics integrating Australian Curriculum must be studying Specialist Methods concurrently.

If students start their study in Specialist Mathematics 2016-2020 course and conclude their course in Mathematical Methods 2016-2020 course then they are awarded the appropriate course in Mathematical Methods.

Students studying a combination of Mathematical Applications *(integrating Australian Curriculum)* course with Mathematical Methods *(integrating Australian Curriculum)* course *or the* Specialist Methods course will be awarded a course in Further Mathematics.

### Arrangements for students continuing study in this course

Students who studied the previous course may undertake any units in this course provided there is no duplication of content.

## Duplication of Content Rules

Students cannot be given credit towards the requirements for a Senior Secondary Certificate for a unit that significantly duplicates content in a unit studied in another course. The responsibility for preventing undesirable overlap of content studied by a student rests with the principal and the teacher delivering the course. Students will only be given credit for covering the content once.

## Guidelines for Delivery

### Program of Learning

A program of learning is what a school provides to implement the course for a subject. This meets the requirements for context, scope and sequence set out in the Board endorsed course. Students follow programs of learning in a college as part of their senior secondary studies. The detail, design and layout of a program of learning are a college decision.

The program of learning must be documented to show the planned learning activities and experiences that meet the needs of particular groups of students, taking into account their interests, prior knowledge, abilities and backgrounds. The program of learning is a record of the learning experiences that enable students to achieve the knowledge, understanding and skills of the content descriptions. There is no requirement to submit a program of learning to the OBSSS for approval. The Principal will need to sign off at the end of Year 12 that courses have been delivered as accredited.

### Content Descriptions

Are all content descriptions of equal importance? No. It depends on the focus of study. Teachers can customise their program of learning to meet their own students’ needs, adding additional content descriptions if desired or emphasising some over others. A teacher must balance student needs with their responsibility to teach all content descriptions. It is mandatory that teachers address all content descriptions and that students engage with all content descriptions.

### Half standard 0.5 units

Half standard units appear on the course adoption form but are not explicitly documented in courses. It is at the discretion of the college principal to split a standard 1.0 unit into two half standard 0.5 units. Colleges are required to adopt the half standard 0.5 units. However, colleges are not required to submit explicit documentation outlining their half standard 0.5 units to the BSSS. Colleges must assess students using the half standard 0.5 assessment task weightings outlined in the framework. It is the responsibility of the college principal to ensure that all content is delivered in units approved by the Board.

## Moderation

Moderation is a system designed and implemented to:

* provide comparability in the system of school-based assessment
* form the basis for valid and reliable assessment in senior secondary schools
* involve the ACT Board of Senior Secondary Studies and colleges in cooperation and partnership
* maintain the quality of school-based assessment and the credibility, validity and acceptability of Board certificates.

Moderation commences within individual colleges. Teachers develop assessment programs and instruments, apply assessment criteria, and allocate Unit Grades, according to the relevant Course Framework. Teachers within course teaching groups conduct consensus discussions to moderate marking or grading of individual assessment instruments and unit grade decisions.

### The Moderation Model

Moderation within the ACT encompasses structured, consensus-based peer review of Unit Grades for all accredited courses over two Moderation Days. In addition to Moderation Days, there is statistical moderation of course scores, including small group procedures, for T courses.

### Moderation by Structured, Consensus-based Peer Review

Consensus-based peer review involves the review of student work against system wide criteria and standards and the validation of Unit Grades. This is done by matching student performance with the criteria and standards outlined in the Achievement Standards, as stated in the Framework. Advice is then given to colleges to assist teachers with, or confirm, their judgments. In addition, feedback is given on the construction of assessment instruments.

### Preparation for Structured, Consensus-based Peer Review

Each year, teachers of Year 11 are asked to retain originals or copies of student work completed in Semester 2. Similarly, teachers of a Year 12 class should retain originals or copies of student work completed in Semester 1. Assessment and other documentation required by the Office of the Board of Senior Secondary Studies should also be kept. Year 11 work from Semester 2 of the previous year is presented for review at Moderation Day 1 in March, and Year 12 work from Semester 1 is presented for review at Moderation Day 2 in August.

In the lead up to Moderation Day, a College Course Presentation (comprised of a document folder and a set of student portfolios) is prepared for each A, T and M course/units offered by the school and is sent into the Office of the Board of Senior Secondary Studies.

### The College Course Presentation

The package of materials (College Course Presentation) presented by a college for review on Moderation Days in each course area will comprise the following:

* a folder containing supporting documentation as requested by the Office of the Board through memoranda to colleges, including marking schemes and rubrics for each assessment item
* a set of student portfolios containing marked and/or graded written and non-written assessment responses and completed criteria and standards feedback forms. Evidence of all assessment responses on which the Unit Grade decision has been made is to be included in the student review portfolios.

Specific requirements for subject areas and types of evidence to be presented for each Moderation Day will be outlined by the Board Secretariat through the *Requirements for Moderation Memoranda* and Information Papers.

### Visual evidence for judgements made about practical performances

It is a requirement that schools’ judgements of standards to practical performances (A/T/M) be supported by visual evidence (still photos or video).

The photographic evidence submitted must be drawn from practical skills performed as part of the assessment process.

Teachers should consult the BSSS website for current information regarding all moderation requirements including subject specific and photographic evidence.

# Appendix B – Course Developers

|  |  |
| --- | --- |
| Name | College |
| James Hassall | Burgmann Anglican School |
| Clare Byrne | Dickson College |
| Kim Tae Kyung | Dickson College |
| Gavin Scales | Gungahlin College |
| Sheikh Faisal | Hawker College |
| Jacqueline Howarth | Hawker College |
| Tim Allan | Trinity Christian School |

# Appendix C – Common Curriculum Elements

Common curriculum elements assist in the development of high-quality assessment tasks by encouraging breadth and depth and discrimination in levels of achievement.

|  |  |  |
| --- | --- | --- |
| Organisers | Elements | Examples |
| create, compose and apply | apply | ideas and procedures in unfamiliar situations, content and processes in non-routine settings |
| compose | oral, written and multimodal texts, music, visual images, responses to complex topics, new outcomes |
| represent | images, symbols or signs |
| create | creative thinking to identify areas for change, growth and innovation, recognise opportunities, experiment to achieve innovative solutions, construct objects, imagine alternatives |
| manipulate | images, text, data, points of view |
| analyse, synthesise and evaluate | justify | arguments, points of view, phenomena, choices |
| hypothesise | statement/theory that can be tested by data |
| extrapolate | trends, cause/effect, impact of a decision |
| predict | data, trends, inferences |
| evaluate | text, images, points of view, solutions, phenomenon, graphics |
| test | validity of assumptions, ideas, procedures, strategies |
| argue | trends, cause/effect, strengths and weaknesses |
| reflect | on strengths and weaknesses |
| synthesise | data and knowledge, points of view from several sources |
| analyse | text, images, graphs, data, points of view |
| examine | data, visual images, arguments, points of view |
| investigate | issues, problems |
| organise, sequence and explain | sequence | text, data, relationships, arguments, patterns |
| visualise | trends, futures, patterns, cause and effect |
| compare/contrast | data, visual images, arguments, points of view |
| discuss | issues, data, relationships, choices/options |
| interpret | symbols, text, images, graphs |
| explain | explicit/implicit assumptions, bias, themes/arguments, cause/effect, strengths/weaknesses |
| translate | data, visual images, arguments, points of view |
| assess | probabilities, choices/options |
| select | main points, words, ideas in text |
| identify, summarise and plan | reproduce | information, data, words, images, graphics |
| respond | data, visual images, arguments, points of view |
| relate | events, processes, situations |
| demonstrate | probabilities, choices/options |
| describe | data, visual images, arguments, points of view |
| plan | strategies, ideas in text, arguments |
| classify | information, data, words, images |
| identify | spatial relationships, patterns, interrelationships |
| summarise | main points, words, ideas in text, review, draft and edit |

# Appendix D – Glossary of Verbs

|  |  |
| --- | --- |
| Verbs | Definition |
| Analyse | Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences |
| Apply | Use, utilise or employ in a particular situation |
| Argue | Give reasons for or against something |
| Assess | Make a Judgement about the value of |
| Classify | Arrange into named categories in order to sort, group or identify |
| Compare | Estimate, measure or note how things are similar or dissimilar |
| Compose | The activity that occurs when students produce written, spoken, or visual texts |
| Contrast | Compare in such a way as to emphasise differences |
| Create | Bring into existence, to originate |
| Critically analyse | Analysis that engages with criticism and existing debate on the issue |
| Demonstrate | Give a practical exhibition an explanation |
| Describe | Give an account of characteristics or features |
| Discuss | Talk or write about a topic, taking into account different issues or ideas |
| Evaluate | Examine and judge the merit or significance of something |
| Examine | Determine the nature or condition of |
| Explain | Provide additional information that demonstrates understanding of reasoning and /or application |
| Extrapolate | Infer from what is known |
| Hypothesise | Put forward a supposition or conjecture to account for certain facts and used as a basis for further investigation by which it may be proved or disproved |
| Identify | Recognise and name |
| Interpret | Draw meaning from |
| Investigate | Planning, inquiry into and drawing conclusions about |
| Justify | Show how argument or conclusion is right or reasonable |
| Manipulate | Adapt or change |
| Plan | Strategize, develop a series of steps, processes |
| Predict | Suggest what might happen in the future or as a consequence of something |
| Reflect | The thought process by which students develop an understanding and appreciation of their own learning. This process draws on both cognitive and affective experience |
| Relate | Tell or report about happenings, events or circumstances |
| Represent | Use words, images, symbols or signs to convey meaning |
| Reproduce | Copy or make close imitation |
| Respond | React to a person or text |
| Select | Choose in preference to another or others |
| Sequence | Arrange in order |
| Summarise | Give a brief statement of the main points |
| Synthesise | Combine elements (information/ideas/components) into a coherent whole |
| Test | Examine qualities or abilities |
| Translate | Express in another language or form, or in simpler terms |
| Visualise | The ability to decode, interpret, create, question, challenge and evaluate texts that communicate with visual images as well as, or rather than, words |

# Appendix E – Glossary for ACT Senior Secondary Curriculum

Courses will detail what teachers are expected to teach and students are expected to learn for year 11 and 12. They will describe the knowledge, understanding and skills that students will be expected to develop for each learning area across the years of schooling.

**Learning areas** are broad areas of the curriculum, including English, mathematics, science, the arts, languages, health and physical education.

A **subject** is a discrete area of study that is part of a learning area. There may be one or more subjects in a single learning area.

**Frameworks** are system documents for Years 11 and 12 which provide the basis for the development and accreditation of any course within a designated learning area. In addition, frameworks provide a common basis for assessment, moderation and reporting of student outcomes in courses based on the framework.

The **course** sets out the requirements for the implementation of a subject. Key elements of a course include the rationale, goals, content descriptions, assessment, and achievement standards as designated by the framework.

BSSS courses will be organised into units. A unit is a distinct focus of study within a course. A standard 1.0 unit is delivered for a minimum of 55 hours generally over one semester.

**Core** units are foundational units that provide students with the breadth of the subject.

**Additional** units are avenues of learning that cannot be provided for within the four core 1.0 standard units by an adjustment to the program of learning.

An **Independent Study unit** is a pedagogical approach that empowers students to make decisions about their own learning. Independent Study units can be proposed by a student and negotiated with their teacher but must meet the specific unit goals and content descriptions as they appear in the course.

An **elective** is a lens for demonstrating the content descriptions within a standard 1.0 or half standard 0.5 unit.

A **lens** is a particular focus or viewpoint within a broader study.

**Content descriptions** refer to the subject-based knowledge, understanding and skills to be taught and learned.

A **program of learning** is what a college develops to implement the course for a subject and to ensure that the content descriptions are taught and learned.

**Achievement standards** provide an indication of typical performance at five different levels (corresponding to grades A to E) following completion of study of senior secondary course content for units in a subject.

ACT senior secondary system **curriculum** comprises all BSSS approved courses of study.

# Appendix F – Glossary for Specialist Methods

## Unit 1

## Functions and graphs

**Asymptote**

A line is an **asymptote** to a curve if the distance between the line and the curve approaches zero as they ‘tend to infinity’. For example, the line with equation is a vertical asymptote to the graph of and the line with equation is a horizontal asymptote to the graph of .

**Binomial distribution**

The expansion is known as the **binomial theorem**. The numbers are called binomial coefficients.

**Completing the square**

The quadratic expression can be rewritten as Rewriting it in this way is called **completing the square**.

**Discriminant**

The **discriminant** of the quadratic expression is the quantity

**Function**

A **function** is a rule that associates with each element in a set a unique element in a set We write to indicate the mapping of to . The set is called the **domain** of and the set is called the **codomain**. The subset of consisting of all the elements is called the **range** of If we write we say that is the **independent variable** and is the **dependent variable.**

**Graph of a function**

The **graph of a function** is the set of all points in Cartesian plane where is in the domain of and

**Quadratic formula**

If with then . This formula for the roots is called the **quadratic formula**.

**Vertical line test**

A relation between two real variables and is a function and for some function if and only if each vertical line, i.e. each line parallel to the axis, intersects the graph of the relation in at most one point. This test to determine whether a relation is, in fact, a function is known as the **vertical line test**.

## Trigonometric functions

**Circular measure** is the measurement of angle size in radians.

**Radian measure**

The **radian measure** of an angle in a sector of a circle is defined by , where is the radius and is the arc length. Thus an angle whose degree measure is has radian measure .

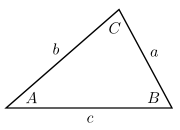
**Length of an arc**

The **length of an arc in a circle** is given by , where is the arc length, is the radius and is the angle subtended at the centre, measured in radians. This is simply a rearrangement of the formula defining the radian measure of an angle.

**Sine rule and cosine rule**

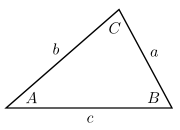
The lengths of the sides of a triangle are related to the sines of its angles by the equations

This is known as the **sine rule**.



The lengths of the sides of a triangle are related to the cosine of one of its angles by the equation

This is known as the **cosine rule**.



**Sine and cosine functions**

In the unit circle definition of cosine and sine, are the coordinates of the point on the unit circle corresponding to the angle

**Period of a function**

The period of a function is the smallest positive number with the property that for all . The functions and both have period and has period

## Counting and Probability

**Pascal’s triangle**

**Pascal’s triangle** is a triangular arrangement of binomial coefficients. The row consists of the **binomial coefficients** for , each interior entry is the sum of the two entries above it, and sum of the entries in the row is

**Conditional probability**

The probability that an event occurs can change if it becomes known that another event occurs. The new probability is known as a **conditional probability** and is written as If has occurred, the sample space is reduced by discarding all outcomes that are not in the event The new sample space, called **the reduced sample space**, is The conditional probability of event is given by .

**Independent events**

Two events are **independent** if knowing that one occurs tells us nothing about the other. The concept can be defined formally using probabilities in various ways: events A and B are independent if , if or if For events and with non-zero probabilities, any one of these equations implies any other.

**Mutually exclusive**

Two events are **mutually exclusive** if there is no outcome in which both events occur.

**Point and interval estimates**

In statistics estimation is the use of information derived from a sample to produce an estimate of an unknown probability or population parameter. If the estimate is a single number, this number is called a **point estimate.** An **interval estimate** is an interval derived from the sample that, in some sense, is likely to contain the parameter.

A simple example of a point estimate of the probability of an event is the relative frequency of the event in a large number of Bernoulli trials. An example of an interval estimate for is a confidence interval centred on the relative frequency

**Relative frequency**

If an event occurs times when a chance experiment is repeated times, the **relative** frequency of is .

## Unit 2

## Exponential functions

**Index laws**

The index laws are the rules: , , , , and , for any real numbers , with

**Algebraic properties of exponential functions**

The algebraic properties of exponential functions are the index laws: , , , , for any real numbers , with

## Arithmetic and Geometric sequences and series

**Arithmetic sequence**

An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence

2, 5, 8, 11, 14, 17, …

is an arithmetic sequence with common difference 3.

If the initial term of an arithmetic sequence is *a* and the common difference of successive members is *d*, then the *n*th term *tn­*, of the sequence, is given by:

*tn* = *a* + (*n* – 1)*d* for *n* ≥ 1

A recursive definition is

*t*1 = *a*, *tn+*1 = *tn*+ *d* where *d* is the common difference and *n* ≥ 1.

**Geometric sequence**

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the **common ratio**. For example, the sequence

3, 6, 12, 24, ...

is a geometric sequence with common ratio 2. Similarly the sequence

40, 20, 10, 5, 2.5, …

is a geometric sequence with common ratio .

If the initial term of a geometric sequence is *a* and the common ratio of successive members is *r*, then the *nth term tn­,* of the sequence, is given by:

*tn* = *arn* – 1 for *n* ≥ 1

A recursive definition is

*t*1 = *a*, *tn*+1= *rtn*for *n* ≥ 1 and where *r* is the constant ratio

**Partial sums of a sequence (Series)**

The sequence of partial sums of a sequence *t*1,…, *ts*, … is defined by

*S­n* = *t*1 + …+ *tn*

**Partial sum of an arithmetic sequence (Arithmetic series)**

The partial sum *Sn* of the first *n* terms of an arithmetic sequence with first term *a* and common difference *d*.

*a*, *a* + *d*, *a* + 2*d*,……, *a* + (*n* – 1)*d,*…

is

*Sn* = (*a* + *tn*) = (2*a* + (*n* – 1)*d*) where *tn* is the *n*th term of the sequence.

The partial sums form a sequence with *Sn*+1 = *Sn*+ *tn* and *S*1 = *t*1

**Partial sums of a geometric sequence (Geometric series)**

The partial sum *Sn* of the first *n* terms of a geometric sequence with first term *a* and common ratio *r*,

*a*, *ar*, *ar*2,……, *arn* – 1,…

is

*Sn* = , *r* ≠ 1.

The partial sums form a sequence with *Sn*+1 = *Sn*+ *tn* and *S*1 = *t*1.

## Introduction to differential calculus

**Gradient (Slope)**

The **gradient** of the straight line passing through points and is the ratio . **Slope** is a synonym for **gradient**.

**Secant**

A **secant** of the graph of a function is the straight line passing through two points on the graph. The line segment between the two points is called a **chord**.

**Tangent line**

The **tangent line** (or simply the **tangent**) to a curve at a given point can be described intuitively as the straight line that "just touches" the curve at that point. At where the tangent meets the curve, the curve has "the same direction" as the tangent line. In this sense it is the best straight-line approximation to the curve at the point .

**Linearity property of the derivative**

The **linearity property of the derivative** is summarized by the equations:

for any constant

and

**Local and global maximum and minimum**

A **stationary** **point** on the graph of a differentiable function is a point where .

We say that is a **local maximum** of the function if for all values of near . We say that is a **global maximum** of the function if for all values of in the domain of .

We say that is a **local minimum** of the function if for all values of near . We say that is a **global minimum** of the function if for all values of in the domain of .

## Unit 3

## The logarithmic function

**Algebraic properties of logarithms**

The algebraic properties of logarithms are the rules: and for any positive real numbers

## Further differentiation and applications

**Euler’s number**

**Euler’s number** is an irrational number whose decimal expansion begins

It is the base of the natural logarithms, and can be defined in various ways including:

and .

**Product rule**

The **product rule** relates the derivative of the product of two functions to the functions and their derivatives.

If then ,

and in Leibniz notation:

**Quotient rule**

The **quotient rule** relates the derivative of the quotient of two functions to the functions and their derivatives

If then

and in Leibniz notation:

**Composition of functions**

If and for functions then is a composite function of

We write

For example, expresses as a composite of the functions and

**Chain rule**

The **chain rule** relates the derivative of the composite of two functions to the functions and their derivatives.

If then

and in Leibniz notation:

**Concave up and concave down**

A graph of is concave up at a point if points on the graph near lie above the tangent at The graph is concave down at if points on the graph near lie below the tangent at

**Point of inflection**

A point on the graph of is a point of inflection if the concavity changes at i.e. points near on one side of lie above the tangent at and points near on the other side of lie below the tangent at

**Second derivative test**

According to the second derivative test, if then is a local maximum of if and is a local minimum if

## Integrals

**Antidifferentiation**

An **anti-derivative**, **primitive** or **indefinite integral** of a function is a function whose derivative is , i.e.

The process of solving for anti-derivatives is called **anti-differentiation**.

Anti-derivatives are not unique. If is an anti-derivative of then so too is the function where is any number. We write to denote the set of all anti-derivatives of The number is called the **constant of integration**. For example, since we can write

**The linearity property of anti-differentiation**

The linearity property of anti-differentiation is summarized by the equations:

for any constant and

for any two functions

Similar equations describe the linearity property of definite integrals:

for any constant and

for any two functions

**Additivity property of definite integrals**

The **additivity property of definite integrals** refers to ‘addition of intervals of integration’:

for any numbers and and any function

**The fundamental theorem of calculus**

**The** **fundamental theorem of calculus** relates differentiation and definite integrals. It has two forms: and

## Unit 4

## Discrete random variables

**Random variable**

A **random variable** is a numerical quantity whose value depends on the outcome of a chance experiment. Typical examples are the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin, and the number of tonnes of wheat produced in Australia in a year.

A **discrete random variable** is one whose possible values are the counting numbers , or form a finite set, as in the first two examples.

A **continuous random variable** is one whose set of possible values are all of the real numbers in some interval.

**Probability distribution**

The **probability distribution** of a discrete random variable is the set of probabilities for each of its possible values.

**Uniform discrete random variable**

A **uniform discrete random variable** is one whose possible values have equal probability of occurrence. If there are possible values, the probability of occurrence of any one of them is .

**Expected value**

The **expected value** of a random variable is a measure of the central tendency of its distribution.

If is discrete, where the are the possible values of and

If is continuous, where is the probability density function of

**Mean of a random variable**

The **mean** of a random variable is another name for its expected value.

**Variance of a random variable**

The **variance** of a random variable is a measure of the ‘spread’ of its distribution.

If is discrete, , where is the expected value.

If is continuous,

**Standard deviation of a random variable**

The **standard deviation** of a random variable is the square root of its variance.

**Effect of linear change**

The **effects of linear changes of scale and origin** on the mean and variance of a random variable are summarized as follows:

If is a random variable and where and are constants, then

and

**Bernoulli random variable**

A Bernoulli random variable has two possible values, namely and . The parameter associated with such a random variable is the probability of obtaining a.

**Bernoulli trial**

A **Bernoulli trial** is a chance experiment with possible outcomes, typically labelled ‘success’ and failure’.

## Continuous random variables and the normal distribution

**Probability density function**

The **probability density function** of a continuous random variable is a function that describes the relative likelihood that the random variable takes a particular value. Formally, if is the probability density of the continuous random variable then the probability that takes a value in some interval is given by

**Uniform continuous random variable**

A **uniform continuous random variable** is one whose probability density function has constant value on the range of possible values of . If the range of possible values is the interval then if and otherwise.

**Triangular continuous random variable**

A **triangular continuous random variable** is one whose probability density function has a graph with the shape of a triangle.

**Quantile**

A **quantile** for a continuous random variable is defined by where

The **median** of is the quantile corresponding to :

## Interval estimates for proportions

**Central limit theorem**

There are various forms of the **Central limit theorem,** a resultof fundamental importance in statistics. For the purposes of this course, it can be expressed as follows:

“If is the mean of independent values of random variable which has a finite mean and a finite standard deviation , then as the distribution of approaches the standard normal distribution.”

In the special case where is a Bernoulli random variable with parameter is the sample proportion and In this case the Central limit theorem is a statement that as the distribution of approaches the standard normal distribution.

**Margin of error**

The **margin of error** of a confidence interval of the form is the half-width of the confidence interval. It is the maximum difference between and if is actually in the confidence interval.

**Level of confidence**

The **level of confidence** associated with a confidence interval for an unknown population parameter is the probability that a random confidence interval will contain the parameter.

# Appendix G – Course Adoption

**Conditions of Adoption**

The course and units of this course are consistent with the philosophy and goals of the college and the adopting college has the human and physical resources to implement the course.

**Adoption Process**

Course adoption must be initiated electronically by an email from the principal or their nominated delegate to [bssscertification@ed.act.edu.au](mailto:bssscertification@ed.act.edu.au). A nominated delegate must CC the principal.

The email will include the **Conditions of Adoption** statement above, and the table below adding the **College** name, and circling the **Classification/s** required.

|  |  |
| --- | --- |
| College: |  |
| Course Title: | Specialist Methods |
| Classification/s: | T |
| Accredited from: | 2016 |
| Framework: | Mathematics 2020 |